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Variable structure fuzzy control using three input variables for reducing motion tracking errors†

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Abstract

Conventional fuzzy controllers for motion tracking utilize generally two input variables (position error and velocity error) to deal with highly nonlinear and time-varying dynamics associated with complex mechanical motion with multi-DOF. This results in some tracking errors at steady state, in general, mainly due to friction existing in mechanical systems. To eliminate the steady-state tracking errors, a variable structure fuzzy control algorithm is proposed using three input variables (position error, velocity error, and integral of position errors) and a switching logic between two inputs and three inputs. Simulation and experimental studies have been conducted to show the validity of the proposed control logic using a direct-drive SCARA manipulator with two degree-of-freedom. It has been shown that the proposed fuzzy control logic has significantly improved motion-tracking performance of the mechanical system when it is applied to complex polygon-tracking in Cartesian space with inverse kinematics and path planning.

Keywords: Cartesian space; Fuzzy control; Motion tracking; Steady-state error; Variable structure control

1. Introduction

Since the dynamics of complex mechanical systems such as robotic manipulators is highly nonlinear and time-varying, a fast and accurate motion control is a challenging problem [1]. The control by way of exact cancellation of all dynamic disturbances such as varying inertia, Coriolis and centrifugal accelerations, and gravitational and frictional disturbances, or the control by way of dealing with all dynamic disturbances as uncertainties, could be very complex and sometimes may not be computationally feasible for fast operating mechanical systems. In this case, fuzzy control is a good candidate since it does not depend on the accurate model of the plant, but is based on heuristics about plant behaviors. Recently, various fuzzy control logics have been tried to be applied to motion control of mechanical systems [2-4].

Fuzzy set theory was first introduced by L. A. Zadeh [5] to deal with imprecise objects. He suggested various possible application fields of fuzzy set theory (by relying on the use of linguistic variables and fuzzy algorithms) where the behaviors of systems are too complex or too ill-defined to admit of precise mathematical analysis [6]. Mamdani [7] first successfully applied fuzzy logic algorithms for the control of a small laboratory steam engine. Thereafter, many applications of fuzzy control were reported in the various engineering fields including chemical processes and consumer products [8-10]. A good summary of the fuzzy logic controls is presented by Lee [11]. Wang [12] viewed fuzzy control theory as a subset of nonlinear control theory. He pointed out better control performance of fuzzy controllers compared to conventional PID controllers is due to the nonlinear behavior of fuzzy controllers.

Motion control problems require faster and more accurate response compared with other industrial

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processes. The applications of fuzzy control algorithms to motion control problems are reported extensively in the literature. Among them, Li and Lau [13] applied fuzzy control to servomotor systems and showed that the performance of the fuzzy controller is better than that of PI controllers in terms of steady state error, settling time and response time through simulation study using two look-up tables for coarse and fine controls. Huang and Tomizuka [14] applied the fuzzy controller to two-dimensional motion tracking control and showed tracking precision and travel time can be improved compared with the system with conventional PD controller by simulation study. Scharf and Mandic [15] applied it to motion control of an indirect drive robot manipulator and showed that step response and tracking performance are often superior to those of conventional PID controllers.

However, the effect of time-varying and nonlinear dynamics in indirect drive is not so severe. Kang et al.[16] applied fuzzy control to motion tracking control of direct drive robotic manipulators, and showed that position tracking performance of fuzzy control is similar to or often better than the one of PID control through simulation and experimental studies.

Recently, Kohn-Rich et al.[2], and Ham et al.[3] designed stable fuzzy controllers for a large class of mechanical systems in the framework of Lyapunov's stability theory. Park et al.[4] proposed a sliding mode control logic with fuzzy adaptive perturbation compensator for parallel manipulators. Adams and Rattan [17] adopted a multi-stage fuzzy PID control scheme with fuzzy switching to reduce steady-state errors. Adams and Rattan's solution shows a nice way to include fuzzy I action in the controller, in which fuzzy PD control output is blended with fuzzy I action before defuzzifying them when fuzzy PD input is in the Zero fuzzy set. Sooraksa et al.[18] applied their fuzzy P^2ID scheme to handlebar control of a bicycle robot.

In this paper, a variable structure fuzzy control logic, a revised algorithm of Kang et al.'s one [16,19], is proposed using three input variables of fuzzy controller (position error, velocity error and integral of position errors) and a switching logic between two inputs and three inputs in order to eliminate position errors existing at steady state in motion control, and it is applied to motion control of a mechanical manipulator.

The present paper has extended the result of the author's previous publication [20] to motion control of Cartesian space additionally using inverse kinematics and Jacobian of the manipulator. The proposed control scheme is different from Adams and Rattan's solution [17] in the sense that two fuzzy control logics, fuzzy PD and fuzzy PID control logic, are designed separately (not multi-stage), and the switching between fuzzy PD and fuzzy PID control occurs according to tracking condition, that is, transient or steady-state condition. The idea of the proposed control scheme is relatively simple but the improvement of actual control performance is significant. This fact has been shown through simulation and experimental studies using a direct drive SCARA manipulator with two degree-of-freedom (DOF) motion.

The paper is organized as follows. Section 2 presents basic terminologies and the structure of the proposed fuzzy control scheme, and Section 3 describes application of the proposed fuzzy control scheme to motion tracking of a mechanical system. In Section 4, computer simulation and experimental results of the proposed fuzzy control are discussed, and concluding remarks are given in Section 5.

2. Variable structure fuzzy control

2.1 Definitions and terminology

A *fuzzy set A* in X is characterized by a *membership function* $\mu(x)$ which associates with each point x in X a real number in the interval [0, 1], with the value 0 representing non-membership and the value 1 representing full membership. The *support* of a fuzzy set A is the crisp set of all points x in X such that $\mu(x) > 0$. A fuzzy set whose support is a single point in X with $\mu(x) = 1$ is referred to as *fuzzy singleton*. Fuzzy set operations and fuzzy logic can be defined in many ways [21]. In the following, we summarize the definitions and terminology used in this paper. The membership function $\mu(x)$ of the *union* $A \cup B$ of two fuzzy sets A and B is pointwise defined for all x in X by $\mu_{A\cup B}(x) = \max\{\mu_A(x), \mu_B(x)\},\ x \in X$, and the membership function $\mu(x)$ of the *intersection* $A \cap B$ is defined for all x in X by $\mu_{A\cap B}(x) = \min\{\mu_A(x), \mu_B(x)\},\ x \in X$, which are originally proposed by Zadeh [5]. *Fuzzy implication* adopted in this paper is *Larsen's product rule* [12] among many other choices, which is defined by $\mu_{A\rightarrow B}(x, y) = \mu_A(x)\mu_B(y)$, $x \in X, y \in Y$. *Fuzzy reasoning* is generally based on a *compositional rule of inference* [6], which can be viewed as an extension of the *modus ponens*. The following *sup-min composition* proposed by Zadeh is used in this paper.

$$B' = A' \circ (A \rightarrow B)$$
$$\mu_{B'}(y) = \sup_{x \in X} \min\{\mu_{A'}(x), \mu_A(x)\mu_B(y)\}$$

A *linguistic variable* is a variable that can take words in natural languages (for example, big, small etc.) as its values. These words are usually labels of fuzzy sets.

2.2 Proposed variable structure fuzzy controller

Position errors due to Coulomb friction, backlash etc., generally remain at steady state in position control of mechanical systems such as robotic manipulators. A conventional fuzzy PD controller using two input variables (position error and velocity error) [16] cannot remove such errors existing at steady state, e.g., at a designation point of the end-effector of a robotic manipulator.

From the idea that steady state errors can be generally reduced if we add an integral action to proportional and derivative control, conventional fuzzy PD controller with two input variables is revised to a fuzzy controller with variable structure between fuzzy PD control and fuzzy PID control logic with three input variables – position error, velocity error, and integral of position errors – and with a switching logic to eliminate position errors existing at steady state in motion control of mechanical systems. Since position errors at steady state are usually small, we integrate position errors only during steady state and use integral values as another input variable of the fuzzy controller to remove small position errors effectively.

The proposed algorithm includes two types of fuzzy control logic: fuzzy PD control and fuzzy PID control logic. If motion tracking is in transient mode, the fuzzy PD control logic is activated, and if it is in a steady-state mode, the fuzzy PID control logic is activated. The switching between fuzzy PD control and fuzzy PID control occurs in a crisp way in this paper for simplicity, although the decision related to the steady state detection could be made in a fuzzy way. That is, if the change of position error during one sampling period is below a threshold value and the position error is below the preset maximum steady state error, then the flag for error integral is turned on and the fuzzy controller with two input variables is switched to the one with three input variables.

If the change of position error during one sampling period becomes again equal to or greater than the threshold value, or the position error becomes equal to or greater than the preset maximum steady state error, then the flag for error integral is turned off and the fuzzy controller with three input variables is restored to the previous fuzzy controller with two input variables.

Even though the stability of the fuzzy logic control system is an important issue, we do not consider the stability problem here and assume the overall system is heuristically stable. For stability issues of fuzzy control systems, refer to the reference papers [2, 3, 18, 22].

A block diagram of the proposed variable structure fuzzy control system is shown in Fig. 1. The sampler and zero-order hold (ZOH) are implemented by timer interrupt and D/A converter (DAC). The position and velocity signals are digital and the fuzzy control action is implemented by using a microprocessor and random access memory (for a look-up table).

The input variables of the fuzzy controller are angular position error e (rad) and angular velocity error $\dot{e}$ (rad/s), and integral of angular position errors $\int e$ (rad $\cdot$ s). The output variable of the fuzzy controller is the control input u (N $\cdot$ m) to the motor driver.

$$e = \textit{position command} - \textit{actual position},$$
$$\dot{e} = \frac{e(kT) - e((k-1)T)}{T},$$
$$\int e = \sum_{i=j}^{k} e(iT)T$$
$$u = f(e, \dot{e}) \quad or \quad u = f(e, \dot{e}, \int e)$$

where T represents sampling time, and i, j and k are positive integers such that k represents present time index and j represents time index at which the steady state condition is detected. In the fuzzy controller, the measured e, $\dot{e}$ and $\int e$ values are scaled to some real values in the interval [-1, 1] and mapped to linguistic variables E, DE and SE by the *fuzzification* operator. The values of linguistic variables are composed of linguistic terms PL, PS, ZO, NS and NL, or

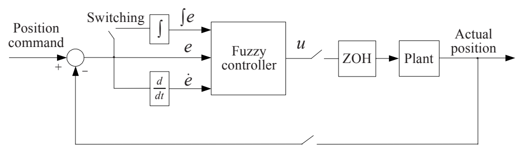
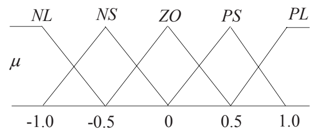
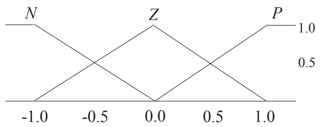


Fig. 1. Block diagram of the proposed fuzzy control system.



(a)



(b)

Fig. 2. Membership functions of fuzzy sets for (a) position error, velocity error, control input, and (b) integral of position errors.

N, *Z* and *P* that are all fuzzy sets.

This linguistic term set forms a *fuzzy partition* of input and output spaces. The *knowledge base* of the fuzzy controller is composed of a *database* and a *rule base.* The database defines membership functions for the above linguistic terms, and the rule base represents fuzzy control rules. Instead of quantizing the universe of discourse of input and output spaces into a finite number of segments (quantization levels), we do a linear scale mapping of position error, velocity error, integral of position errors and control input with a value in [-1, 1], and so keep infinite resolutions in the fuzzification and defuzzification operations (that is, we do not lose some information from the given data at these stages).

Membership functions of fuzzy sets (linguistic terms) and fuzzy control rules have a considerable effect on control performance. Fig. 2 shows the membership functions of the fuzzy sets for position error e, velocity error $\dot{e}$, control input u, and integral of position errors $\int e$.

We use membership functions with the same triangular shapes and same *support* sizes, such as Fig. 2. However, we also can use membership functions with the same triangular shapes but different support sizes. With membership functions with different support sizes, we can emphasize that the control near zero is more important than that of the other portions.

A *fuzzy control rule* is a *fuzzy conditional statement* (IF-THEN statement) in which the antecedents and the consequent are associated with fuzzy concepts (linguistic terms). One example of a fuzzy control rule is as follows:

R_1: If *E* is *ZO* and *DE* is *ZO* and *SE* is *N*, then *U* is *NS*.
R_2: If *E* is *PS* and *DE* is *NS* and *SE* is *Z*, then *U* is *ZO*.

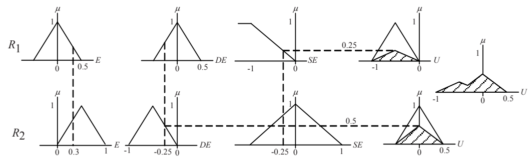


Fig. 3. Graphical representation of the inference mechanism.

These rules are implemented by *fuzzy relation* R_1 = {(*ZO* and *ZO* and *N*) $\rightarrow$ *NS*} and R_2 = {(*PS* and *NS* and *Z*) $\rightarrow$ *ZO*} where '*and*' and '$\rightarrow$' are processed by min operation (intersection) and Larsen's product rule, respectively.

Fuzzy reasoning (or approximate reasoning) is done by the sup-min composition operator. As an illustration, consider the above two fuzzy control rules R_1 and R_2. Let the inputs be *fuzzy singletons*, namely, *E'* = 0.3, *DE'* = -0.25 and *SE'* = -0.25. Then the *inference mechanism* adopted here can be represented graphically as shown in Fig. 3.

Defuzzification is a mapping from fuzzy control actions into nonfuzzy (crisp) control actions. Among many defuzzification strategies, we adopt the *center average method* [12].

The calculation time consumed for fuzzy inference and fuzzification is usually considerable (e.g., a few milliseconds). To save the calculation time in real-time control, we calculate *U* for the standard *E*, *DE* and *SE* values in advance, and form a three-dimensional *look-up table*. Since the number of input variables could be two or three according to tracking situations (transient or steady state), we need two-dimensional and three dimensional look-up tables together. Roughly speaking, a two-dimensional look-up table is necessary for transient condition, and a three-dimensional look-up table is for steady state condition. These two look-up tables are used alternately as time passes.

3. Implementation of the proposed control scheme to motion tracking

The mechanical system with two DOF motion considered in this paper is shown in Fig. 4. This system is actually a two axes SCARA manipulator moving in a horizontal plane. The control system has two inputs and four outputs. Two inputs are two motor-driving signals, and four outputs are two position signals and two velocity signals of two motors coming from resolvers. Furthermore, this mechanical system has highly nonlinear and time-varying dynamics including

Fig. 4. Photograph of the 2 DOF mechanical system used in simulation and experimental studies.

Coriolis acceleration and coupling effects between axes. This manipulator is driven by two direct drive motors (Megatorque motors). This system was designed and integrated in our laboratory. The range of input voltages to driver units is -10 volts to 10 volts.

Torque generated by the Megatorque motor is proportional to input voltage. Maximum torques of the base motor and the upper motor are 147 N·m and 9.8 N·m, respectively. The resolver is attached to each Megatorque motor to measure angular position and velocity. Resolver signals are converted to phase *A* and *B* quadrature signals by RDC (Resolver to Digital Converter) in the Driver Unit, and these quadrature signals are counted by each counter board. The resolutions of feedback signals of the base motor and the upper motor are 38,400 counts/rev and 25,600 counts/rev, respectively. Maximum speeds of the base motor and the upper motor are 3 rev/sec and 4.5 rev/sec, respectively.

A dynamic model of the robot is not necessary for designing a fuzzy controller, but we develop one for simulation purpose. The dynamics of an *n*-DOF, rigid link, direct drive manipulator can be generally described as follows [23, 24]:

$$\mathbf{M}(\theta(t))\dot{\omega}(t) + \mathbf{v}(\theta(t),\omega(t)) + \mathbf{g}(\theta(t)) + \mathbf{f}(\omega(t),\tau(t)) = \tau(t)$$

where θ is an angular position vector of joints, ω is an angular velocity vector of joints, τ is an input torque vector supplied by actuators, $\mathbf{M}$ is a symmetric and positive definite generalized inertia matrix, $\mathbf{v}$ is a vector due to Coriolis and centrifugal forces, $\mathbf{f}$ is viscous and Coulomb friction torque vector, and $\mathbf{g}$ is a gravitational torque vector. For the two-axes direct-drive manipulator considered in this paper, each term in the above equation is given as follows:

Table 1. Scale mapping of the position error, velocity error, integral of position errors and control input.

Scaled values	e_1 rad	e_2 rad	$\int e_1$ rad·s	$\int e_2$ rad·s	$\dot{e}_1, \dot{e}_2$ rad/s	u_1 N·m	u_2 N·m
-1.0	-0.30	-0.10	-3.0e-4	-8.0e-4	-4.0	-145	-9.8
-0.5	-0.15	-0.05	-1.5e-4	-4.0e-4	-2.0	-72.5	-4.9
0	0	0	0	0	0	0	0
0.5	0.15	0.05	1.5e-4	4.0e-4	2.0	72.5	4.9
1.0	0.30	0.10	3.0e-4	8.0e-4	4.0	145	9.8

$$\theta(t) = \begin{bmatrix} \theta_1(t) \\ \theta_2(t) \end{bmatrix}, \quad \omega(t) = \begin{bmatrix} \omega_1(t) \\ \omega_2(t) \end{bmatrix}, \quad \tau(t) = \begin{bmatrix} \tau_1(t) \\ \tau_2(t) \end{bmatrix}$$

$$\mathbf{M}(\theta) = \begin{bmatrix} m_{11} & m_{12} \\ m_{12} & m_{22} \end{bmatrix}, \quad \begin{aligned} m_{11} &= p_3 + 2p_1\cos\theta_2 \\ m_{12} &= p_2 + p_1\cos\theta_2 \\ m_{22} &= p_2 \end{aligned}$$

$$\mathbf{v}(\theta,\omega) = \begin{bmatrix} -\omega_2(2\omega_1 + \omega_2)p_1\sin\theta_2 \\ \omega_1^2 p_1 \sin\theta_2 \end{bmatrix}$$

$$\mathbf{f}(\omega,\tau) = \begin{bmatrix} 0.24\times\omega_1 + f_1 \\ 0.035\times\omega_2 + f_2 \end{bmatrix},$$

$$f_i = \begin{cases} T_i \ \mathrm{sign}\omega_i \ \text{ if } |\omega_i| > 0 \\ T_i \ \mathrm{sign}\tau_i \text{ if } \omega_i = 0 \text{ and } |\tau_i| > T_i \\ \tau_i \text{ if } \omega_i = 0 \text{ and } |\tau_i| \le T_i \end{cases}$$

$$T_1 = 3.2 N\cdot m, \ T_2 = 0.17 N\cdot m$$
$$\mathbf{g}(\theta) = 0$$

where

$$p_1 = 0.123 + 0.08M_p$$
$$p_2 = 0.138 + 0.0625M_p + I_p$$
$$p_3 = 1.676 + 0.165M_p + I_p$$
$$M_p = 0 \quad or \quad 3.76kg$$
$$I_p = 0 \quad or \quad 0.012kg\cdot m^2$$

M_p is the mass of payload and I_p is the moment of inertia of payload. All the values above are represented in SI units.

For this mechanical system, the fuzzy controller proposed in the previous section is applied. First we define a linear scale mapping for position error, velocity error, integral of position errors and control input by values in [-1, 1]. Table 1 shows a summary of the linear scale mapping where subscripts 1 and 2 represent the base motor and the upper motor, respectively.

In the table, the maximum value of the control input is the same with the maximum torque that Megatorque motor can generate. These maximum values are the values that we allow at the corresponding variables. In some sense, these values act like the inverses of the proportional and derivative gain in PD control. However, the difference of the fuzzy controller and PD controller is that the fuzzy controller acts in a nonlinear fashion.

The whole fuzzy control rules of the fuzzy controller for this mechanical system are shown in Table 2. Linguistic terms in Table 2 are defined in Fig. 2.

In Table 2, the integral term (*SE*) is activated (i.e., summed) only when the steady state condition is detected from the way proposed in Section 2.2 in which the flag for error integral is used. At the steady state condition, position error (*E*) and velocity error (*DE*) are both *ZO*. In this way, the number of fuzzy control rules is just increased from 25 to 27 even if the number of input variables is increased from 2 to 3.

After fuzzy inference and defuzzification processes presented in the previous section, the actual control input u is obtained by multiplying the scaling factor 145 (for base motor) or 9.8 (for upper motor). According to control input u, actual torque τ is generated by the motor.

For the present mechanical system, one three-dimensional look-up table instead of two look-up tables can be generated and used from the given fuzzy control rules in Table 2. In view of the required memory size and accuracy of the fuzzy inference, 21x21 *U* values in the look-up table are generated for *SE* = 0, and 11x11x21 *U* values are generated for the other *SE* values. For *SE* = 0, the values of *E* and *DE* are divided into interval of 0.1 from –1 to 1. For the other *SE* values, the values of *E* and *DE* are divided into the same interval from -0.5 to 0.5, and *SE* into the same interval from –1 to 1.

In this way, we save the required memory size up to 70 % compared to the memory size required for three-dimensional look-up table (21x21x21) with intervals of 0.1 for three input variables. The 21x21 table for *SE* = 0 can be considered as the two-dimensional look-up table that is necessary for a transient condition of motion tracking. For a specific position error, velocity error and integral of position errors, control input is calculated through a linear interpolation by using look-up tables as shown in Fig. 5. For specific input values, the interpolated fuzzy controller output *U* is given as follows (using the notations in Fig. 5):

$$U = \left(1-\frac{\Delta x}{x}\right)\left(1-\frac{\Delta y}{y}\right)\left(1-\frac{\Delta z}{z}\right)U_1 + \left(1-\frac{\Delta x}{x}\right)\left(1-\frac{\Delta y}{y}\right)\frac{\Delta z}{z}U_2 + \left(1-\frac{\Delta x}{x}\right)\frac{\Delta y\Delta z}{yz}U_3 + \left(1-\frac{\Delta x}{x}\right)\frac{\Delta y}{y}\left(1-\frac{\Delta z}{z}\right)U_4 + \frac{\Delta x\Delta y\Delta z}{xyz}U_5 + \frac{\Delta x}{x}\left(1-\frac{\Delta y}{y}\right)\left(1-\frac{\Delta z}{z}\right)U_6 + \frac{\Delta x\Delta y}{xy}\left(1-\frac{\Delta z}{z}\right)U_7 + \frac{\Delta x\Delta z}{xz}\left(1-\frac{\Delta y}{y}\right)U_8$$

Table 2. Fuzzy control rules for position control of the robotic manipulator.

If input variables are E and DE (transient condition), or if input variables are E, DE and SE with SE=Z (steady-state condition), then U are

E \ DE	*NL*	*NS*	*ZO*	*PS*	*PL*
NL	*NL*	*NL*	*NL*	*NS*	*ZO*
NS	*NL*	*NL*	*NS*	*ZO*	*PS*
ZO	*NL*	*NS*	*ZO*	*PS*	*PL*
PS	*NS*	*ZO*	*PS*	*PL*	*PL*
PL	*ZO*	*PS*	*PL*	*PL*	*PL*

If E = ZO, DE = ZO and SE = P (steady-state condition), then U = PS.

If E = ZO, DE = ZO and SE = N (steady-state condition), then U = NS.

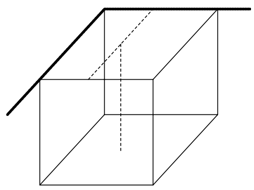


Fig. 5. Interpolation in the three dimensional lookup table.

As the tracking condition (transient or steady-state) of the manipulator is changed, the proposed control scheme switches the fuzzy PD control logic to the fuzzy PID control logic and vice versa according to the switching logic explained in the previous section.

4. Simulation and experimental results

For the 2 DOF mechanical system described in the previous section, digital simulation and experimental studies have been conducted to show that the proposed variable structure fuzzy controller reduces steady-state position errors significantly compared to conventional fuzzy controllers. One simple chosen task is to drive each axis of the manipulator independently in joint space without considering total end-effector motion, which was shown in the author's previous Korean publication [20]. Here the results are summarized briefly again for comparison purposes.

For this task, the desired trajectory for both axes is given by the seventh-order polynomial as shown in Fig. 6. That is, the task is to drive both motors at the same time with the trajectory given in Fig. 6. The overlapped curve with the desired trajectory in Fig. 6 is an experimental result for the upper axis when the proposed fuzzy control is applied for this task.

Since the distinction between the desired trajectory and the actual trajectory is difficult in Fig. 6, we plot position errors in a magnified scale to evaluate control performance clearly. Fig. 7 (a) shows the simulation result, and Fig. 7 (b) shows the experimental result for position error of the base axis when the proposed fuzzy controller (denoted by "Proposed Fuzzy" in the figures) and the conventional fuzzy controller (denoted by "Old Fuzzy") are applied with 3.76 kg payload. Here the conventional fuzzy controller implies the fuzzy controller with two input variables. From Fig. 7 (a) and Fig. 7 (b), we see that the proposed fuzzy control reduces position errors more than 80 % (in magnitude) at steady state compared to those of the conventional fuzzy control, and has similar position tracking errors to those of the conventional fuzzy control at transient response intervals. The simulation and experimental results for the steady state position errors are summarized in Table 3.

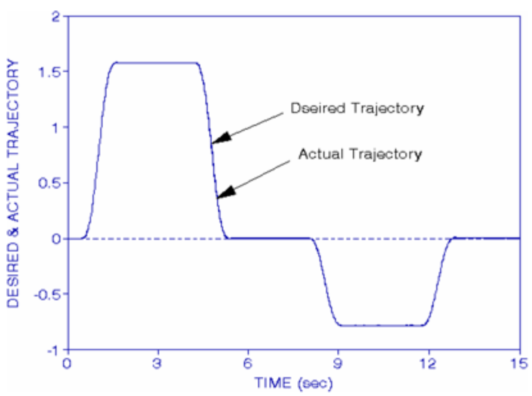
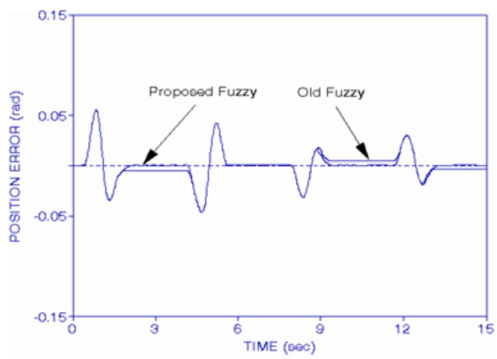


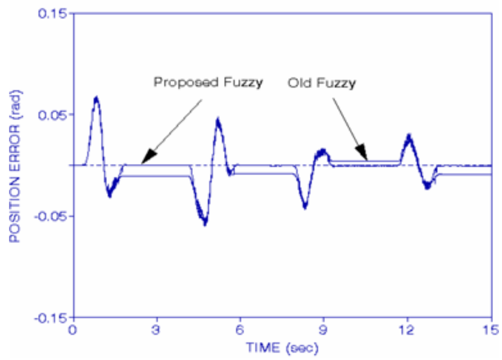
Fig. 6. Desired and actual position trajectories of the upper motor when the proposed fuzzy control is applied without payload (experiment).

Table 3. Simulation and experiment results for the steady state position errors.

		Convent. fuzzy (A)	Proposed fuzzy (B)	B/A
Simul.	Base axis (no payload)	-2.5e-3 rad	0.5e-4 rad	2 %
	Upper axis (no payload)	-2.0e-3 rad	1.6e-4 rad	8 %
	Base axis (3.76 kg)	5.4e-3 rad	-1.1e-4 rad	2 %
	Upper axis (3.76 kg)	0.3e-3 rad	0.3e-4 rad	10 %
Exper.	Base axis (no payload)	-9.8e-3 rad	6.8e-4 rad	7 %
	Upper axis (no payload)	2.0e-3 rad	3.0e-4 rad	15 %
	Base axis (3.76 kg)	-10.8e-3 rad	1.1e-4 rad	1 %
	Upper axis (3.76 kg)	1.2e-3 rad	2.0e-4 rad	17 %



(a)



(b)

Fig. 7. Position tracking errors of the base axis when the proposed and the conventional fuzzy control are applied with 3.76 *kg* payload. (a) simulation, (b) experiment.

However, the motion of the manipulator should be controlled in Cartesian space in order to be applied to actual factory operations. Thus, the previous results are extended to a complex task to drive the manipulator in Cartesian space using inverse kinematics and path planning. The selected Cartesian space task is to draw various polygons on the table with the manipulator.

In the simulation, the controller is realized in a discrete time domain, but the manipulator is realized as a continuous time model (using a 4th-order Runge-Kutta algorithm). Sampling time T is set to 5 ms in the simulation and experiment. In the experiment, elapsed time for executing the interrupt service routine within one sampling interval was measured to be about 0.3 ms. To keep sampling intervals exact in the experiments, we used timer interrupts. Programming for simulations and experiments was coded with C language.

To realize tracking control in Cartesian space, we have to solve the inverse kinematics problem on-line. The solution of the inverse kinematics problem and manipulator Jacobian are obtained as closed-forms for this SCARA manipulator as follows:

$$\theta_2 = \tan^{-1}\left(\frac{\sqrt{1-(x^2+y^2-a_1^2-a_2^2)^2/(4a_1^2a_2^2)}}{(x^2+y^2-a_1^2-a_2^2)/(2a_1a_2)}\right)$$

$$\theta_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{a_2\sin\theta_2}{a_1+a_2\cos\theta_2}\right)$$

$$J = \begin{bmatrix} -a_1\sin\theta_1 - a_2\sin(\theta_1+\theta_2), & -a_2\sin(\theta_1+\theta_2) \\ a_1\cos\theta_1 + a_2\cos(\theta_1+\theta_2), & a_2\cos(\theta_1+\theta_2) \end{bmatrix}$$

where x, y are Cartesian coordinates, θ_1, θ_2 are joint coordinates, a_1, a_2 are link lengths, and J is the manipulator Jacobian.

We have selected polygons (square, triangle and hexagon) in Fig. 10 in order to see improvement of tracking performance of the proposed fuzzy controller. The Cartesian coordinate at every sampling instant is generated via path planning. Velocity command profiles of each side of the polygons are programmed to be an LSPB (linear segments with parabolic blends) type as shown in Fig. 8. Specifically, during 1/6 interval of one side of the polygons, motion is constantly accelerated, and during the next 2/3 interval of the side, motion has constant maximum velocity, and then during the last 1/6 interval of the side, motion is constantly decelerated. In other words, the velocity profile forms a trapezoidal shape in Cartesian space, not in time domain.

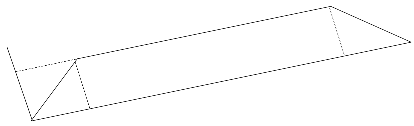


Fig. 8. Velocity command profile of each side of the polygons; LSPB type between two points A and B in space.

Fig. 9 shows the whole block diagram of the control system in Cartesian space. In the figure, $\mathbf{x}_d$ represents the desired position vector in Cartesian coordinate, and $\boldsymbol{\theta}_d$ does the desired position vector in joint space.

Fig. 10 shows results of simulations and experiments for motion tracking of square, triangle and hexagon when the proposed variable structure fuzzy control is applied. Tracking control performance is evaluated by using integral of squared error (ISE) that is defined as summation of the squared value of position error between desired position and actual position in Cartesian space at every sampling instant. The unit of ISE in Fig. 10 is $mm^2 \cdot s$.

Fig. 10(a) is an experimental result for square path tracking with maximum velocity 200 mm/s in the LSPB, and Fig. 10(b) is an experimental result when maximum velocity is doubled to be 400 mm/s. In this case, maximum angular speeds of the motors are within 20 % of the maximum angular speeds that can be generated by the motors. From a comparison of (a) and (b), we see that the ISE value is increased as tracking speed is increased. Fig. 10(c) is the simulation result at the same operating condition with (b). Fig. 10(b) and (c) show similar tracking patterns. Fig. 10(d) is the magnified experimental result at the corner of the square of (a). Fig. 10(d) shows significant improvement of the proposed fuzzy controller for corner tracking compared to the conventional fuzzy controller with two input variables.

Fig. 10(e) and (f) are experimental results to show a comparison of control performance of the proposed fuzzy control and classical PID control. We tuned PID control gains roughly via Ziegler-Nichols tuning-rules and then finely tuned. However, PID control gains may possibly be more tuned to have less ISE value, but it takes time and is a tedious job.

Fig. 10(g) is an experimental result when impact force is exerted on the side of the upper link during

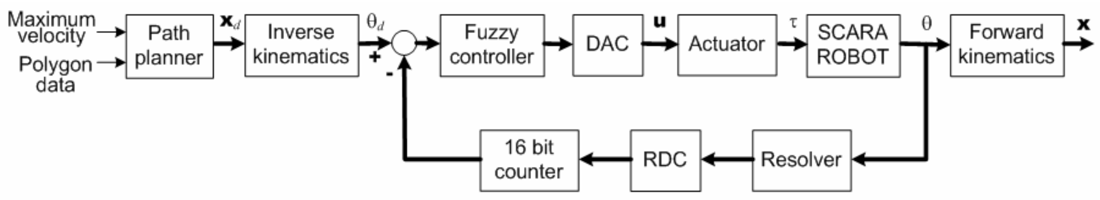


Fig. 9. The whole block diagram of the control system in Cartesian space.

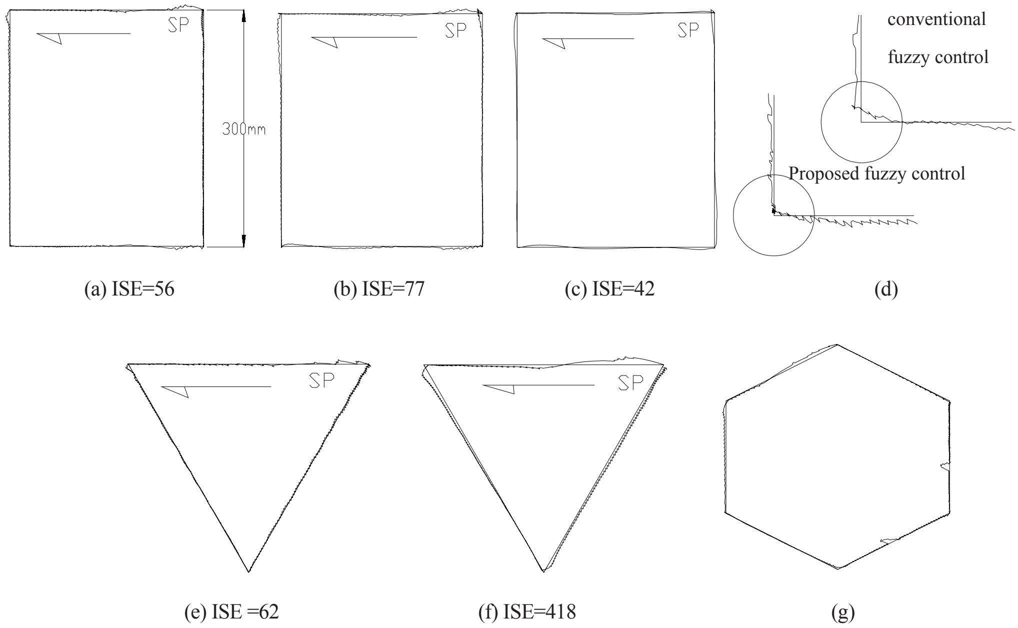


Fig. 10. Position tracking for polygon paths. (a) experiment with maximum velocity = 200 mm/s, (b) experiment with maximum velocity = 400 mm/s, (c) simulation with maximum velocity = 400 mm/s, (d) comparison of the conventional fuzzy control and the proposed fuzzy control, (e) experiment with the proposed fuzzy controller, (f) experiment with PID controller, (g) experiment with impact disturbances.

operation, in order to see the response for external disturbance. We hit the link with a thick book. The experimental result (g) shows that the stability of the system with the proposed fuzzy control logic is maintained for external disturbances with relatively big magnitudes. This robust stability comes, we believe, from the fact that designing the proposed fuzzy controller does not depend on precise knowledge of actual dynamics and uncertainties of the system.

5. Conclusions

When a conventional fuzzy PD controller with two input variables (position error and velocity error) is applied to motion tracking control of mechanical systems with complex dynamics and nonlinearities, position errors usually remain at steady state mainly due to friction. To reduce these errors, we have proposed a revised fuzzy controller with variable structure between fuzzy PD control and fuzzy PID control and with a switching logic between two fuzzy controllers according to tracking conditions (transient or steady state), and applied it to motion control of a mechanical system in Cartesian space.

When the proposed variable structure fuzzy control is applied to two DOF mechanical systems with complex dynamics and nonlinearities as shown in Fig. 4, the position errors at steady state have been decreased more than 80 % (in magnitude) compared to ones of the conventional fuzzy control. Moreover, the proposed fuzzy control logic has improved tracking control performance significantly, especially, at corner trackings of polygons in Cartesian space.

With the proposed implementation method of the proposed fuzzy controller, we saved required memory size up to 70 % compared to memory size required for conventional three-dimensional look-up table.

Conclusively, the idea of the proposed variable structure fuzzy control scheme is relatively simple but the improvement of actual control performance has been significant in both simulations and experiments.

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